IV Workshop de Matemática, Estatística e Computação Aplicadas à Indústria

Application of Numerical Methods in the Placement of Boreholes in Mineral Exploration Projects

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1 Introduction

Mineral exploration represents a fundamental stage in the mining cycle, dedicated to the identification and evaluation of deposits with economic potential. This process comprises several successive phases, ranging from the preliminary recognition of promising areas based on public data and geological mappings to the execution of drillings aimed at the three-dimensional modeling of mineral bodies and the accurate estimation of resources and reserves.

Among the main challenges of this stage, the high cost of drilling campaigns stands out, as they account for the largest share of investment in the exploratory phase. In this context, the development of strategies capable of strategically guiding borehole placement becomes increasingly relevant, as it helps to reduce uncertainties and avoid the allocation of resources in areas with low mineral potential. The combined use of geospatial information with mathematical and computational modeling techniques thus emerges as a promising alternative to support decision-making processes.

In light of this scenario, the present work proposes the integrated application of numerical methods and Principal Component Analysis (PCA) to the study of existing drilling data, with the objective of identifying zones with a higher probability of mineralized continuity. This approach aims to provide a basis for planning new drillings, enhancing the accuracy in target definition, and contributing to a more efficient use of resources in mineral exploration.

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2 Materials and Methods

2.1 Database

The data used in this study were obtained from Nexa Resources' *Exploration Reports* [3–5], referring to drilling campaigns conducted in 2024 for the *Aripuanã brownfield* project. The dataset comprises 5,255 borehole records containing geographic coordinates, total depth, and mineralized intervals, from which the mineralized length was derived.

For confidentiality, the coordinates were normalized to the interval $[0, 1]$ while preserving spatial proportions. Exploratory analysis revealed a higher drilling density within $x \in [0.075, 0.25]$ and $y \in [0.70, 0.95]$, indicating a region of stronger sampling and mineral continuity. The original and restricted spatial distributions are shown in Figure 1, which guided the definition of the main study area and subsequent modeling.

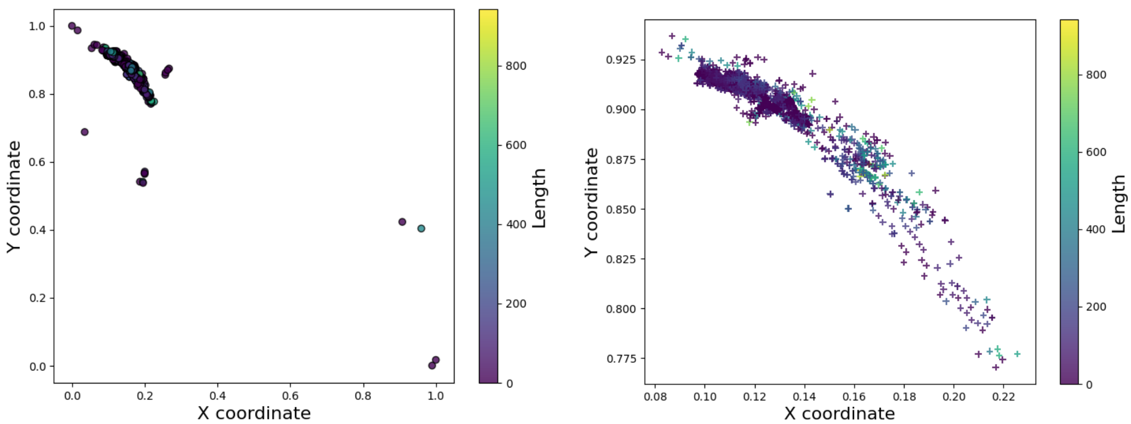

Figura 1: Left: overall distribution of boreholes and their respective mineralized depths. Right: delimited area of interest for detailed analysis.

2.2 Computational Modeling and Graph Construction

The boreholes were represented on a normalized Cartesian plane, where each point corresponds to a drilling position. To estimate the spatial distribution of mineralization, a connectivity structure among points was built.

An initial approach using Euclidean distance with a fixed threshold showed high execution time (≈94 s) and low connection density, proving inadequate for the dataset size. Alternatively, Delaunay Triangulation [2] was applied, forming triangles that maximize internal angles. This method greatly reduced execution time (≈0.05 s) and produced a consistent spatial mesh used in subsequent modeling stages.

2.3 Mathematical Formulation and Numerical Solution

The triangulated mesh was represented as a graph, whose connectivity is described by the *adjacency matrix* A, where $A_{ij} = 1$ if two vertices are connected and $A_{ij} = 0$ otherwise. From

A, the *degree matrix* D was computed as a diagonal matrix with elements $D_{ii} = \sum_j A_{ij}$, and the *graph Laplacian* was obtained as $L = D - A$, which captures the spatial relationships among neighboring vertices and enforces smoothness in the interpolated field.

To ensure that the numerical solution preserves the observed values at borehole locations, a *penalty matrix* P was introduced. This diagonal matrix applies a large weight ($\alpha = 10^7$) to vertices with known measurements, constraining them to remain fixed during the solution process. The resulting system is expressed as:

$$(L + P)x_{to} = Pb_{to}, \quad (L + P)x_{from} = Pb_{from}, \tag{1}$$

where b_{to} and b_{from} denote the known upper and lower limits of mineralized zones.

Two numerical methods were evaluated: Singular Value Decomposition (SVD) and the Conjugate Gradient (CG) method. Both achieved similar accuracy, but the CG method converged substantially faster (4.8 s versus 183 s for SVD), and was therefore adopted to generate the mineral continuity map.

2.4 Spatial Analysis of New Drillings Using PCA

Based on the obtained continuity map, Principal Component Analysis (PCA) was applied with the objective of identifying directions of greater spatial variability and supporting the definition of new drilling points. Two complementary strategies were evaluated: (i) the isolated application of PCA, responsible for generating new points around the centroid of the dataset, thereby expanding the coverage of the study area; and (ii) the integration of PCA into the connectivity graph, which incorporates the spatial structure of the data and prioritizes regions with lower centrality, corresponding to less-sampled areas.

Both approaches followed defined geometric constraints, including minimum distance between drillings and non-negative depth. The proposed locations were analyzed in relation to the map generated by the Conjugate Gradient method, allowing the overlap of regions with a higher probability of mineral continuity with zones of lower sampling density.

3 Results

The application of the approaches based on Principal Component Analysis (PCA) and on the integration between PCA and graphs resulted in distinct spatial configurations for the new drilling points, revealing complementary characteristics between the evaluated methods.

The isolated application of PCA produced a more dispersed distribution of the proposed drillings, expanding the coverage of the study area and facilitating the identification of new mineralized zones. In contrast, the combined PCA+graph approach yielded a higher concentration of points in regions of low connectivity, effectively filling gaps in the drilling network and contributing to the refinement of the spatial model. Both strategies complied with the established geometric criteria, including the minimum spacing between drillings and consistency with the local topography.

The two-dimensional projections shown in Fig. 2 provide a complementary visualization of the suggested locations. The isolated PCA application (Fig. 2, left) reveals a broader dispersion, whereas the integration with the graph (Fig. 2, right) highlights the overlap of the new drillings with the mineral continuity map obtained using the Conjugate Gradient method, emphasizing the regions with the highest continuity potential.

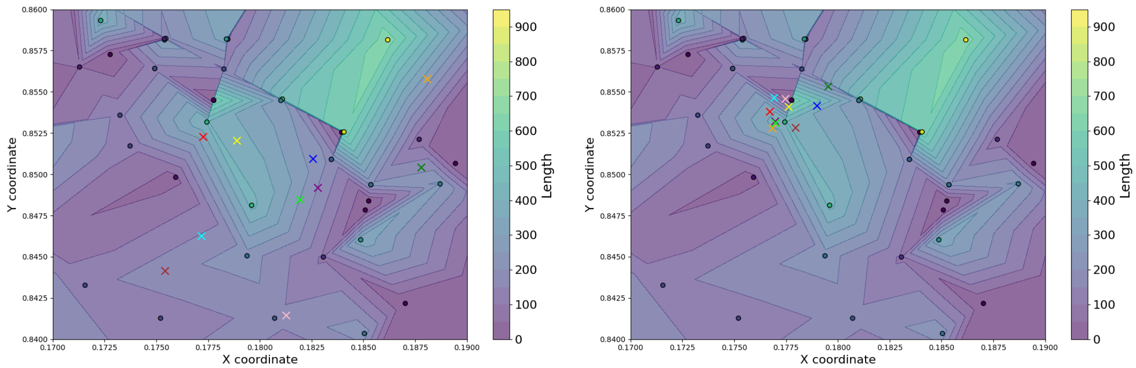

Figura 2: Left: two-dimensional projection of the drilling points proposed by the isolated PCA application. Right: projection obtained using the PCA+graph approach, overlaid on the mineral continuity map estimated with the Conjugate Gradient method.

The comparative analysis of the results confirms that the isolated application of PCA is more suitable for the initial phases of mineral exploration, in which the goal is to expand sampling coverage and identify new areas of interest. Conversely, the combination of PCA and graph proved more appropriate for confirmation and detailing stages, allowing for greater control over the spatial distribution of drillings and reducing uncertainties associated with deposit continuity. This methodological differentiation provides valuable support for planning drilling campaigns at distinct stages of geological investigation.

Referências

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